

Semi-inclusive Processes: phenomenology of Transverse Momentum Dependent distributions

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Transverse Spin Phenomena and Their Impact on QCD a Workshop in Honor of Gary Goldstein's 70th Birthday

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Transverse Momentum Dependent distributions

Spin structure of spin-1/2 nucleon is described by 8 TMDs. Each of them depend on two independent variables x and \mathbf{k}_\perp .

N \ q	U	L	T
U	f_1		h_1^\perp
L		g_1	h_{1L}^\perp
T	f_{1T}^\perp	g_{1T}	h_1 h_{1T}^\perp

time-reversal odd

Plot courtesy of B. Musch

Kotzinian 1995;

Mulders, Tangerman 1995; Boer and

Mulders 1997; Bacchetta et al 2007

T-odd TMDs – Sivers and Boer-Mulders functions survive due to Final State Interactions.

Quark-quark Correlator and TMDs



$$\Phi_{ij}(p, P, S) = \int \frac{d^4\xi}{(2\pi)^4} e^{ik \cdot \xi} \langle P, S | \bar{\psi}_j(0) \mathcal{W}(0, \xi | n^-) \psi_i(\xi) | P, S \rangle$$

Mulders, Tangerman 95; Goeke, Metz, Schlegel 05, Bacchetta et al 07

Twist-2 decomposition (= leading terms in $P^+ = xp^+$ expansion) contains **8 functions**:

$$\Phi^{[\gamma^+]}(x, \mathbf{p}_T, S) = f_1(x, \mathbf{p}_T^2) - \frac{\epsilon_T^{ij} \mathbf{p}_{Ti} S_{Tj}}{M} f_{1T}^\perp(x, \mathbf{p}_T^2)$$

$$\Phi^{[\gamma^+ \gamma_5]}(x, \mathbf{p}_T, S) = S_L g_{1L}(x, \mathbf{p}_T^2) - \frac{\mathbf{p}_{Ti} S_{Ti}}{M} g_{1T}^\perp(x, \mathbf{p}_T^2)$$

$$\Phi^{[i\sigma^{i+} \gamma_5]}(x, \mathbf{p}_T, S) = S_T^i h_1 + S_L \frac{\mathbf{p}_T^i}{M} h_{1L}^\perp - \frac{\mathbf{p}_T^i \mathbf{p}_T^j - 1/2 \mathbf{p}_T^2 g_T^{ij}}{M^2} S_{Tj} h_{1T}^\perp - \frac{\epsilon_T^{ij} \mathbf{p}_{Tj}}{M} h_1^\perp$$

Kotzinian 95; Mulders, Tangerman 96; Barone, Drago, Ratcliffe 02; Bacchetta et al 07; Anselmino et al 06

Polarised Semi Inclusive Deep Inelastic Scattering

Asymmetry in γ^*p cm frame of $lp^\uparrow \rightarrow \ell' h X$

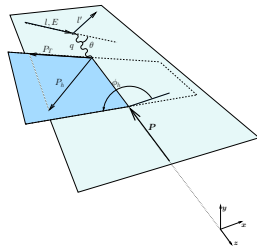
TMD functions can be studied in asymmetries

$$A_{UT} = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{\frac{1}{2}(d\sigma^\uparrow + d\sigma^\downarrow)}$$

Unpolarised electron beam, Transversely polarised proton. Azimuthal dependence on Φ_h and Φ_S singles out different combinations.

Contributions at leading twist

$$d\sigma^\uparrow - d\sigma^\downarrow \propto \underbrace{f_{1T}^\perp \otimes d\hat{\sigma} \otimes D_{h/q} \sin(\phi_h - \phi_S)}_{\text{Sivers effect}} + \underbrace{h_1 \otimes \Delta\hat{\sigma}^\uparrow \otimes H_1^\perp \sin(\phi_h + \phi_S)}_{\text{Collins effect}} + \dots$$



Kotzinian 1995;

Mulders, Tangerman 1995; Boer and

Mulders 1997; Bacchetta et al 2007

The fundamental distributions of partons inside a nucleon

Unpolarised Distribution

$f_1(x)$ or $q(x)$



Distribution of unpolarised partons in an unpolarised nucleon.

Well known

Helicity Distribution

$g_1(x)$ or $\Delta q(x)$

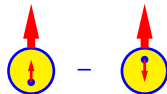


Distribution of longitudinally polarised partons in a longitudinally polarised nucleon.

Known

Transversity Distribution

$h_1(x)$ or $\Delta_T q(x)$



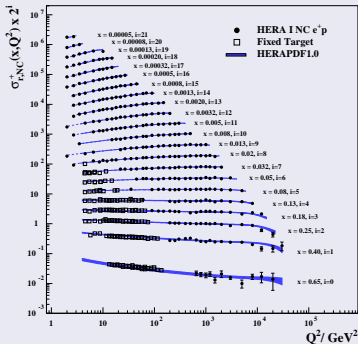
Distribution of transversely polarised quarks in a transversely polarised nucleon.

Little known!
HERMES and COMPASS
experimental measurements

The fundamental distributions of partons inside a nucleon

Unpolarised

H1 and ZEUS



D
p
nucleon.

Well known

Helicity Distribution

$\Delta q(x)$

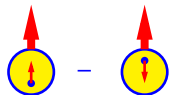


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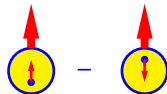


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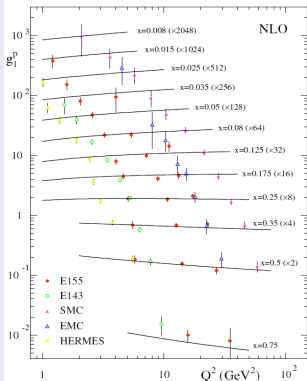
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$f_1(x)$ or $q(x)$



Distribution of unpolarised partons in an unpolarised nucleon.

Well known

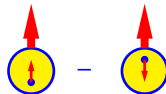


partons in a longitudinally polarised nucleon.

Known

Transversity Distribution

$h_1(x)$ or $\Delta_T q(x)$



Distribution of transversely polarised quarks in a transversely polarised nucleon.

Little known!

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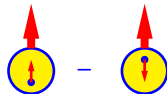


Distribution of longitudinally polarised partons in a longitudinally polarised nucleon.

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$h_1(x)$ or $\Delta_T q(x)$



Distribution of transversely polarised quarks in a transversely polarised nucleon.

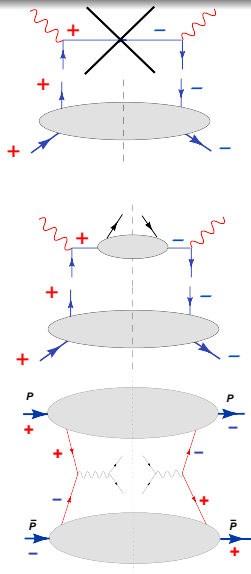
Little known!
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TRANSVERSITY

Transversity cannot be studied in DIS as QED and QCD interactions conserve helicity up to corrections $\mathcal{O}(m_q/E)$.

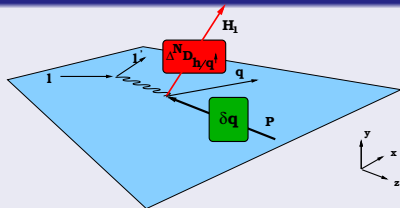
Transversity can be measured if coupled with another chiral-odd function. This can be done in Semi Inclusive DIS (SIDIS), quark fragments into unpolarised hadron. It couples to so called Collins Fragmentation function that describes how a polarised quark fragments into unpolarised hadron.

Golden channel to study transversity is proton - antiproton double spin asymmetry at GSI
 $A_N \propto h_{q/P}(x)h_{\bar{q}/\bar{P}}(x)$.



How to measure transversity? SIDIS and e^+e^- annihilation

SIDIS $lN \rightarrow l'H_1X$



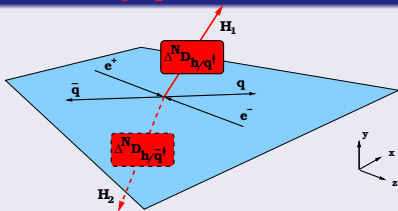
Collins effect gives rise to azimuthal Single Spin Asymmetry

$$\begin{array}{c} \uparrow \\ \circ \\ \uparrow \end{array} - \begin{array}{c} \uparrow \\ \circ \\ \downarrow \end{array} = \Delta_T q(x, Q^2)$$

$$\begin{array}{c} \uparrow \\ \circ \\ \uparrow \end{array} - \begin{array}{c} \uparrow \\ \circ \\ \downarrow \end{array} = \Delta^N D_{h/q^\uparrow}(z, Q^2)$$

J. C. Collins, *Nucl. Phys.* **B396** (1993) 161

$e^+e^- \rightarrow H_1 H_2 X$



Collins effect gives rise to azimuthal asymmetry, q and \bar{q} Collins functions are present in the process:

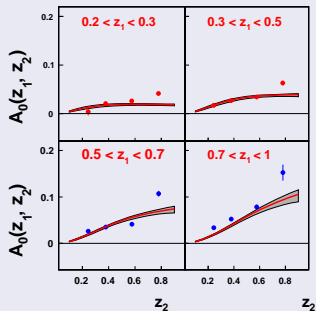
$$\Delta^N D_{h/q^\uparrow}(z_1, Q^2)$$

$$\Delta^N D_{h/\bar{q}^\uparrow}(z_2, Q^2)$$

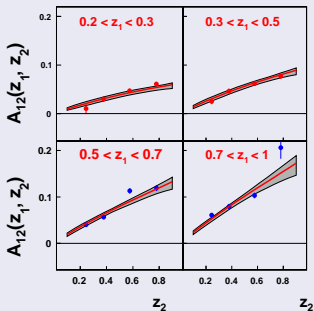
D. Boer, R. Jacob and P. J. Mulders *Nucl. Phys.* **B504** (1997) 345

Description of the data e^+e^-

BELLE $\cos(2\varphi_0)$



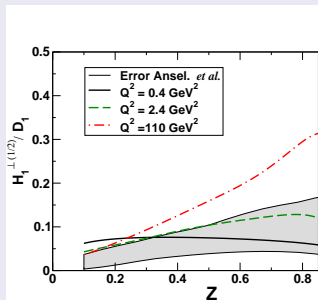
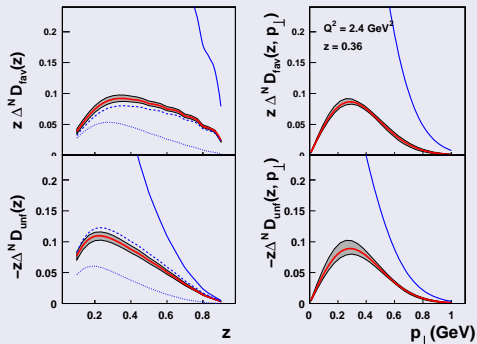
BELLE $\cos(\varphi_1 + \varphi_2)$



$e^+e^- \rightarrow \pi\pi X$, $\sqrt{s} = 10.58$ GeV

Belle, K. Abe et al., Phys. Rev. Lett. 96 (2006) 232002

Collins fragmentation function



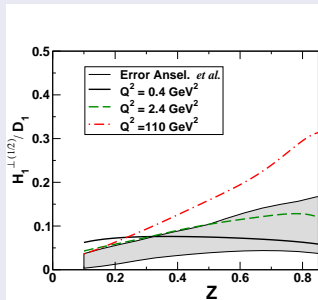
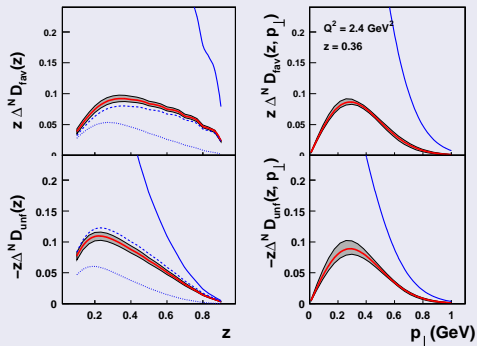
compared (left) to Ref. [1] (dashed line), Ref. [2] (dotted line)

[1] A. V. Efremov, K. Goeke, and P. Schweitzer, Phys. Rev. **D73**, 094025 (2006).

[2] W. Vogelsang and F. Yuan, Phys. Rev. **D72**, 054028 (2005).

and to (right) A, Bacchetta, L. Gamberg, Gary Goldstein, A. Mukherjee Phys. Lett. **B**, 659 (2008).

Collins fragmentation function



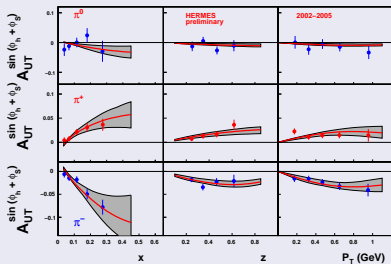
$$\Delta^N D_{fav} = \Delta^N D_{u/\pi^+} > 0$$

$$\Delta^N D_{unfav} = \Delta^N D_{u/\pi^-} < 0$$

$$|\Delta^N D_{fav}| \simeq |\Delta^N D_{unfav}|$$

Experimental data

HERMES $A_{UT}^{\sin(\phi_h + \phi_S)}$

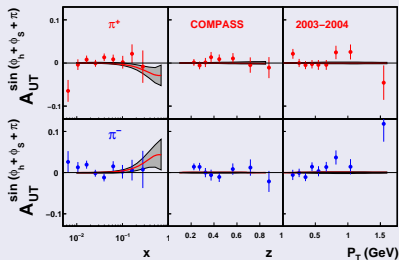


$ep \rightarrow e\pi X$, $p_{lab} = 27.57$ GeV.

HERMES, M. Dieffenthaler, (2007), arXiv:0706.2242

COMPASS, M. Alekseev et al., (2008), Phys.Lett.B673:127-135,2009

COMPASS $A_{UT}^{\sin(\phi_h + \phi_S + \pi)}$

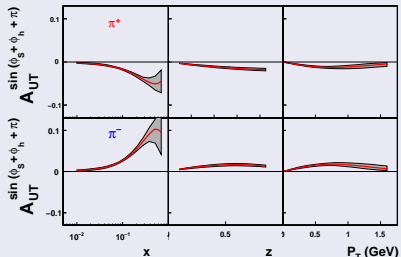


$\mu D \rightarrow \mu\pi X$, $p_{lab} = 160$ GeV

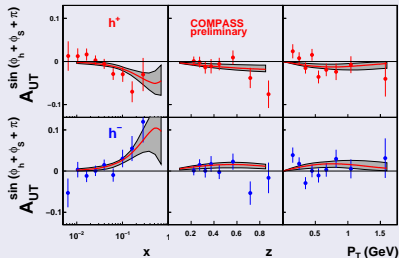
Description of the data

Predictions for COMPASS operating on PROTON target

COMPASS $A_{UT}^{\sin(\phi_h + \phi_S + \pi)}$



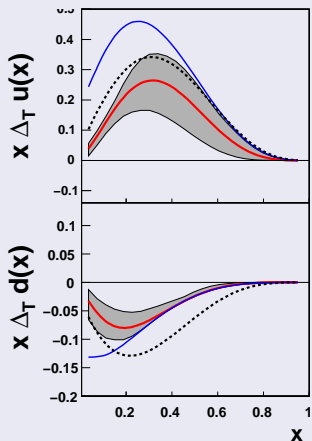
COMPASS $A_{UT}^{\sin(\phi_h + \phi_S + \pi)}$



Comparison with preliminary
COMPASS data arXiv:0808.0086

Anselmino et al 2009

Transversity vs. helicity



- 1 Solid red line – transversity distribution

$$\Delta_T q(x)$$

this analysis at $Q^2 = 2.4 \text{ GeV}^2$.

- 2 Solid blue line – Soffer bound

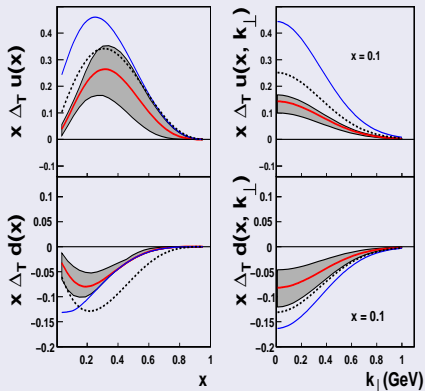
$$|\Delta_T q(x)| < \frac{q(x) + \Delta q(x)}{2}$$

GRV98LO + GRSV98LO

- 3 Dashed line – helicity distribution

$$\Delta q(x)$$

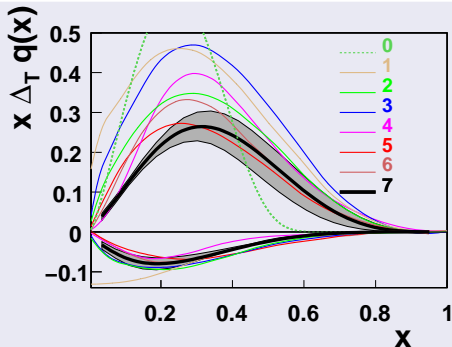
GRSV98LO



- This is the extraction of **transversity** from existing experimental data. Anselmino et al 2009
- $\Delta_T u(x) > 0$ and $\Delta_T d(x) < 0$
- $|\Delta_T q(x)| < |\Delta q(x)|$.
- JLab @ 12 GeV will provide wider region of x for tensor charge extraction.

Transversity, comparison with models

New extraction is close to most models.

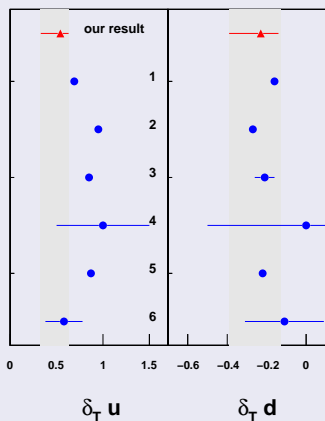


- ① Barone, Calarco, Drago PLB 390 287 (97)
- ① Soffer et al. PRD 65 (02)
- ② Korotkov et al. EPJC 18 (01)
- ③ Schweitzer et al. PRD 64 (01)
- ④ Wakamatsu, PLB B653 (07)
- ⑤ Pasquini et al., PRD 72 (05)
- ⑥ Cloet, Bentz and Thomas PLB 659 (08)
- ⑦ Anselmino et al 2009.

Tensor charges

$$\delta_T q = \int_0^1 dx (h_{1q} - h_{1\bar{q}}) = \int_0^1 dx h_{1q}$$

$$\delta_T u = 0.54_{-0.22}^{+0.09}, \delta_T d = -0.23_{-0.16}^{+0.09} \text{ at } Q^2 = 0.8 \text{ GeV}^2$$



- 1 Quark-diquark model:
Cloet, Bentz and Thomas
PLB **659**, 214 (2008), $Q^2 = 0.4 \text{ GeV}^2$
- 2 CQSM:
M. Wakamatsu, PLB **653** (2007) 398.
 $Q^2 = 0.3 \text{ GeV}^2$
- 3 Lattice QCD:
M. Gockeler et al.,
Phys.Lett.B627:113-123,2005 ,
 $Q^2 = 4 \text{ GeV}^2$
- 4 QCD sum rules:
Han-xin He, Xiang-Dong Ji,
PRD 52:2960-2963,1995, $Q^2 \sim 1 \text{ GeV}^2$
- 5 Constituent quark model:
B. Pasquini, M. Pincetti, and S. Boffi,
PRD72(2005)094029 and PRD76(2007)034020,
 $Q^2 \sim 0.8 \text{ GeV}^2$
- 6 Spin-flavour SU(6) symmetry
L. Gamberg, G. Goldstein,
Phys.Rev.Lett.87:242001,2001 $Q^2 \sim 1 \text{ GeV}^2$

Sivers effect

The azimuthal asymmetry $A_{UT}^{\sin(\phi_h - \phi_S)}$ arises due to Sivers function (Sivers 90)

$$f_{q/p^\uparrow}(x, \mathbf{k}_\perp) = f_{q/p}(x, \mathbf{k}_\perp) - f_{1T}^\perp(x, \mathbf{k}_\perp) \frac{\mathbf{S}_T \cdot (\hat{\mathbf{P}} \times \mathbf{k}_\perp)}{M}$$

Spin sum rule Jaffe, Manohar 90:

$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma + \Delta G + \langle L_z^{q,\bar{q}} \rangle + \langle L_z^G \rangle$$

EMC result on $\Delta\Sigma = \sum_{q,\bar{q}} \Delta q \simeq 0.3$ triggered so called "Spin crisis" – only 30% of the spin of the proton is carried by quarks.

Leader, Anselmino 'A Crisis In The Parton Model: Where, Oh Where Is The Proton's Spin?'
Z.Phys.C41:239,1988

$\mathbf{S}_T \cdot (\hat{\mathbf{P}} \times \mathbf{k}_\perp)$ – correlation between the spin (\mathbf{S}_T) and parton motion (\mathbf{k}_\perp) implies non zero contribution orbital angular momentum.

Data are available from HERMES and COMPASS. u and d Sivers functions are non zero.

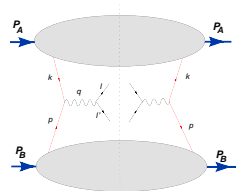
Sivers function: process dependence

Sivers function [Sivers 1990](#) can be measured in both SIDIS and DY processes.

$$f_{q/P^\uparrow}(x, \mathbf{k}_\perp, S) = f_1(x, \mathbf{k}_\perp^2) - \frac{S_T \cdot (\hat{P} \times \mathbf{k}_\perp)}{M} f_{1T}^\perp(x, \mathbf{k}_\perp^2)$$

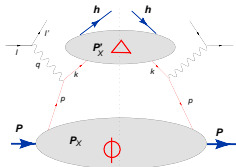
Drell Yan $A^\uparrow B \longrightarrow l^+ l^- X$

$$A_{UT}^{sin(\phi_\gamma - \phi_S)} \sim f_{1T}^{\perp DY}(x, k_\perp) \otimes f_{\bar{q}/B}(x, p_\perp)$$



SIDIS $\ell P^\uparrow \longrightarrow \ell' h X$

$$A_{UT}^{sin(\phi_H - \phi_S)} \sim f_{1T}^{\perp SIDIS}(x, k_\perp) \otimes D_{h/q}(z, p_\perp)$$



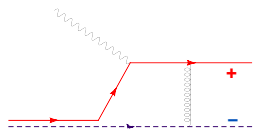
Modified universality

Sivers function is process dependent. Collins 2002

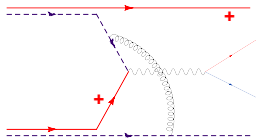
$$f_{1T}^{\perp DY} = -f_{1T}^{\perp SIDIS}$$

Let's consider a simple model of Final State Interactions as in Brodsky, Hwang, Schmidt 2002,

proton = quark⁺ + antiquark⁻



SIDIS - attractive



DY - repulsive

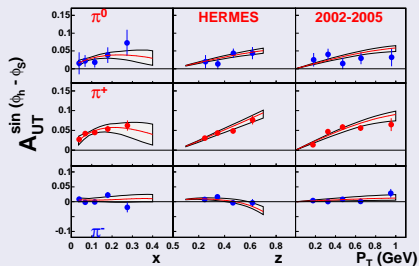
- Experimental test of this relation is fundamental for our understanding of the origin of the correlation between parton angular momentum and the spin of the proton and the gauge link formalism itself.

Experimental DY data are not available, experiments are planned.

HERMES and COMPASS DATA.

HERMES

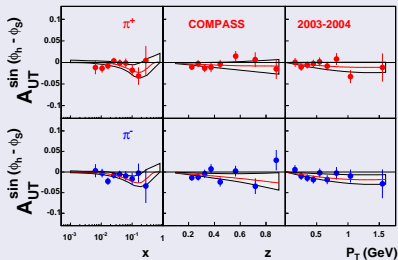
$ep \rightarrow e\pi X$, $p_{lab} = 27.57$ GeV.



M. Anselmino et al 2009

COMPASS

$\mu D \rightarrow \mu\pi X$, $p_{lab} = 160$ GeV.



M. Anselmino et al 2009

$$lp^\uparrow \rightarrow l\pi^+ X \simeq \Delta^N u \otimes D_{u/\pi^+} > 0$$

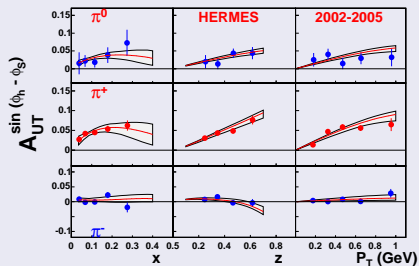
$$lp^\uparrow \rightarrow l\pi^- X \simeq 4\Delta^N u \otimes D_{u/\pi^-} + \Delta^N d \otimes D_{d/\pi^-} \simeq 0$$

$$lD^\uparrow \rightarrow l\pi^+ X \simeq (\Delta^N u + \Delta^N d) \otimes D_{u/\pi^+} \simeq 0$$

HERMES and COMPASS DATA.

HERMES

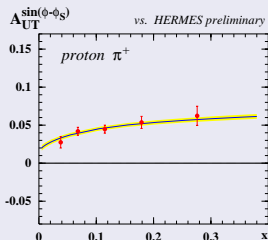
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M. Anselmino et al 2009

HERMES

$ep \rightarrow e\pi X$, $p_{lab} = 27.57$ GeV.



Arnold et al 2008

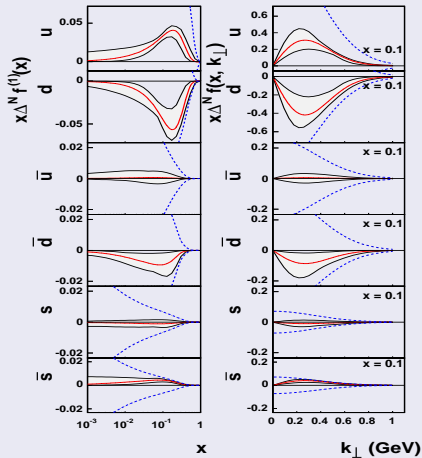
$$lp^\uparrow \rightarrow l\pi^+ X \simeq \Delta^N u \otimes D_{u/\pi^+} > 0$$

$$lp^\uparrow \rightarrow l\pi^- X \simeq 4\Delta^N u \otimes D_{u/\pi^-} + \Delta^N d \otimes D_{d/\pi^-} \simeq 0$$

$$lD^\uparrow \rightarrow l\pi^+ X \simeq (\Delta^N u + \Delta^N d) \otimes D_{u/\pi^+} \simeq 0$$

Sivers functions

$$\Delta^N f_q^{(1)}(x) \equiv \int d^2 \mathbf{k}_\perp \frac{k_\perp}{4m_p} \Delta^N f_{q/p\uparrow}(x, k_\perp) = -f_{1T}^{\perp(1)q}(x).$$



Sivers functions for *u*, *d* and *sea* quarks are extracted from **HERMES** and **COMPASS** data. $\Delta^N f_u > 0$, $\Delta^N f_d < 0$, first hints on nonzero sea quark Sivers functions.

Sivers function comparison with models

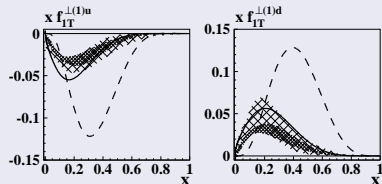
There is a number of model calculations of Sivers function

Light-cone quark model Barbara Pasquini and Feng Yuan 2010

Diquark model Alessandro Bacchetta et al 2010, Leonard Gamberg, Gary Goldstein, and Marc Schlegel 2008 etc

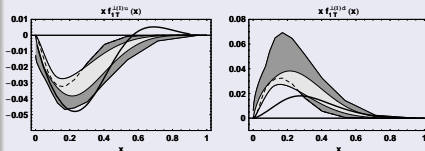
MIT bag model Feng Yuan 2003, H. Avakian, A.V. Efremov, P. Schweitzer, F. Yuan 2010 etc

Pasquini and Yuan 2010



Pasquini and Yuan arXiv:1001.5398

Alessandro Bacchetta et al 2010

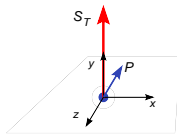
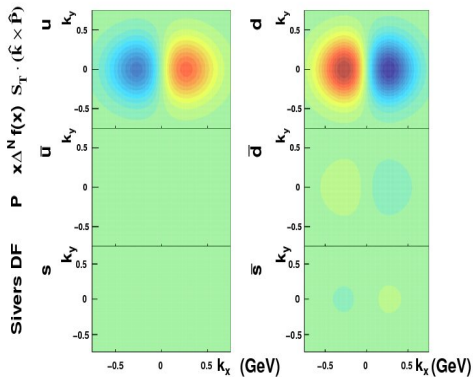


Bacchetta et al arXiv:1003.1328

Reasonable agreement of the extracted Sivers functions Anselmino et al 2009 and Collins et al 2005 and model calculations.

Three dimensional picture of the proton

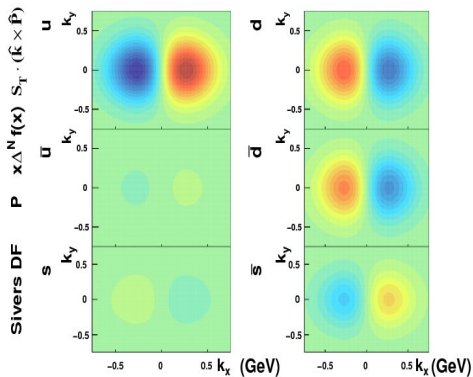
The proton moves along $-Z$ direction (into the screen) and S_T is along Y .



This is the three dimensional view of the proton as “seen” by the virtual photon.
 Red color – more quarks. Blue color – less quarks. Distributions of quarks are not symmetrical and shifted due to final state interactions.
 $x = 0.2$

Three dimensional picture of the proton

The proton moves along $-Z$ direction (into the screen) and S_T is along Y .



Sivers functions for u , d and sea quarks are extracted from **HERMES** and **COMPASS** data. Red color – more quarks. Blue Color – less quarks. Sivers functions is a left – right asymmetry of quark distribution. $x = 0.01$

More information on sea quarks. Future Electron Ion Collider and JLab will contribute.

CONCLUSIONS

- **8** Transverse Momentum Dependent functions describe spin structure of the proton at twist-2.
- Spin Asymmetries are used to study TMDs experimentally.
- T-odd TMDS: Sivers function has *modified universality*, it changes sign from SIDIS to DY.
- HERMES, COMPASS, JLAB, RHIC, and BELLE provide lots of experimental data for TMD extraction.
- Model and lattice QCD calculations of TMDs are possible and match well with TMDs extracted from the experimental data.
- Future facilities such as JLab @ 12 GeV, Electron Ion Collider and GSI will contribute to unravel three dimensional structure of the proton.

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HAPPY BIRTHDAY
GARY!

will